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THE SHEAR ENVELOPE IN CREEP OF BEAMS OF THIN-WALLED OPEN CROSS SECTION

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Abstract—The paper generalizes for creep the concept of the shear center for straight thin-walled beams of open cross section. It is shown that the shear center is replaced by the envelope of a family of straight lines—the torsion-free axes—such that when the plane of bending passes through such a straight line the cross section will suffer no torsion. A method is presented for the calculation of this envelope.

Assuming that the stress–strain rate law is of the power type, it is shown that this envelope is a function of the exponent n of the power law. Since the value of n varies with the temperature, a change in the temperature requires a translation of the plane of loading in order to avoid torsion. Omission of this translation introduces additional shearing stress due to torsion of an order of magnitude equal to that of the shearing stress due to bending. The theory is applied to a semicircular cross section and the computations were made on an IBM 7094-7040 computer.

INTRODUCTION

THE purpose of this paper is to generalize the concept of the shear center for a straight-thin-walled beam of open cross section under conditions of creep. It will be shown that for creep, in order to avoid torsion, instead of a shear center we have to use a shear envelope the size and location of which depend on the creep law. The shear envelope will be determined for a thin-walled beam of semicircular cross section, and the engineering significance of the replacement of the shear center by a shear envelope will be assessed.

The standard assumptions of the elementary theory of bending of beams will be adopted for creep as it was done by several authors in discussing the determination of the shear center when Hooke's law is valid; see, for example, Timoshenko [1], Maillart [2] and Weber [3]. It will be assumed, therefore, that plane cross sections remain plane and that the longitudinal stresses are those which follow from the assumption of pure bending. These longitudinal stresses are used in order to determine the shearing stresses from equilibrium considerations alone. It will be assumed that both the longitudinal stresses and the shearing stresses are uniformly distributed over the thickness of the wall and that the shearing stresses are parallel to the middle surface of the wall. Finally, it will be assumed that the deformation of the cross section is not large enough to influence appreciably the value of the stresses.

Consider the cross section in Fig. 1 where the point 0 is the centroid of the cross section and the point S is the shear center when Hooke's law is valid. For linear elasticity all neutral axes pass through the point 0 and to each neutral axis NN corresponds a line PP passing through S such that when the plane of loading passes through PP the neutral axis will be NN and the cross section will suffer no torsion. The line PP will be called a *torsion free* axis.

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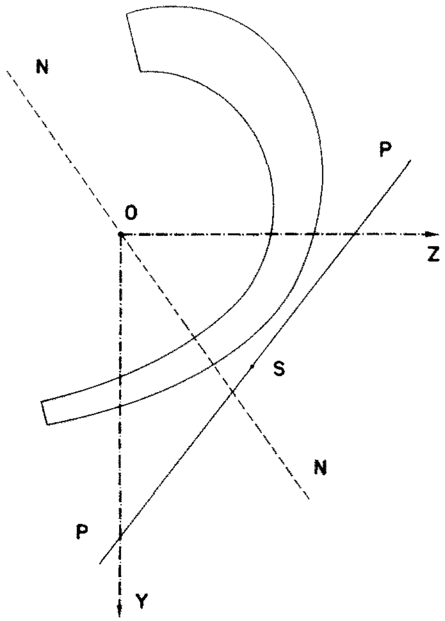


FIG. 1.

Johnson and Mellor [4] calculated the position of the torsionfree axis for symmetrical bending of a member of thin-walled cross section using a power type stress–strain law.

It was shown by Phillips and Donath [5] that for a nonlinear stress–strain law and for creep the neutral axes do not pass through a given point but they form a family of lines; the envelope of these lines was determined in [5] for a power type stress–strain law. It was also shown by Phillips [6] that the torsionfree axes *PP*, for a nonlinear stress–strain law form a family of torsionfree axes which do not pass through the same point (see also the analysis by Patel *et al.* [7]). This family of axes has an envelope which will be determined in this paper.

Consider two planes of loading passing through torsionfree axes. In linear elasticity these two planes will be generated from each other by a rotation since all torsionfree axes pass through the same point 0. In creep, however, it is necessary, in addition to rotation, to introduce a translation in order to generate the one plane of loading from the other. It follows then that a rotation of the plane of loading will introduce torsional loading of the beam unless the rotation is accompanied by a translation of the plane of loading by an amount which is a function of the rotation. In addition, suppose that the stress–strain rate law is of the power type

$$\sigma = (\operatorname{sign} \dot{\varepsilon}) A |\dot{\varepsilon}|^{n}. \tag{1}$$

Then the family of torsionfree axes will be shown to be a function of the exponent n. The value of n varies with the temperature as indicated in Odqvist [8] and Hult [9]. Consequently, a change in the temperature will require a change in the amount of translation of the plane of loading which, according to the discussion above, is required to avoid torsion. It will be shown that omission of this translation will introduce additional shearing stresses (due to torsion) of an order of magnitude which may be equal to that of the shearing stresses due to bending.

It is, therefore, of considerable interest to determine the envelope of the torsionfree axes so that the significance of the lack of a single shear center can be assessed.

This paper is divided into two parts. In the first part we develop the general theory, while in the second part the theory is applied to a semicircular cross section and, on this basis, the significance of the lack of a single shear center is discussed. The computations were made on an IBM 7094–7040 computer.

GENERAL THEORY

Consider a thin-walled beam of arbitrary cross section as in Fig. 2. The system of coordinates XYZ is selected such that the X axis is parallel to the centroidal line of the beam. The assumption that plane cross sections remain plane during creep leads to the expression

$$\dot{\varepsilon} = \dot{\alpha}y + \dot{\beta}z + \dot{\gamma} \tag{2}$$

for the strain rate, where the dots denote derivatives with respect to time. If $\sigma = f(\dot{\varepsilon})$ is the stress rate of strain law of creep, we obtain

$$\sigma = f(\dot{\alpha}y + \dot{\beta}z + \dot{\gamma}) \tag{3}$$

for the stress distribution.

It will be assumed that axial forces are not applied and that normal and shearing stresses are uniformly distributed over the width B of the cross section, and, finally, that the shearing stresses are parallel to the corresponding tangent to the middle line of the cross section.

From Fig. 3 it follows that the equation of equilibrium for an element of the cross section is

$$\frac{\partial}{\partial s}[B(s)\tau(s)] = -\frac{\partial \sigma(s)}{\partial x}B(s) \tag{4}$$

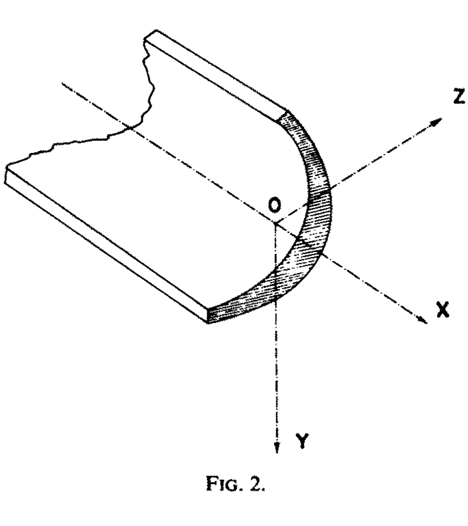


FIG. 2.

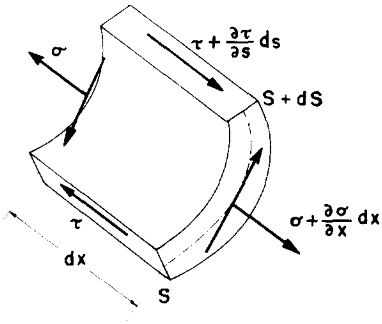


FIG. 3.

where τ is the shearing stress and s is the length of the middle line of the cross section, which varies from $s = 0$ at one end to $s = L$ at the other end. Integrating equation (4), we obtain

$$\tau(s) = -\frac{1}{B(s)}\int_0^s B(s)\frac{\partial\sigma(s)}{\partial x}\,\mathrm{d}s + f_1(x). \tag{5}$$

As the edges of the cross section are free from shearing stresses we obtain $\tau(0) = 0$. Hence the constant of integration in equation (5) $f_1(x) = 0$, so that

$$\tau(s) = -\frac{1}{B(s)}\int_0^s B(s)\frac{\partial\sigma(s)}{\partial x}\,\mathrm{d}s. \tag{6}$$

These shearing stresses produce a shear flow which, in turn, gives rise to a torque T about the X axis:

$$T = \int_{s=0}^{s=L} B(s)\tau(s)R(s)\,\mathrm{d}s \tag{7}$$

where $R(s)$ is the normal distance from the X axis to the shearing stress $\tau(s)$, (Fig. 4). Hence,

$$T = -\int_{s=0}^{s=L} R(s)\int_0^s B(s)\frac{\partial\sigma(s)}{\partial x}\,\mathrm{d}s\,\mathrm{d}s. \tag{8}$$

In order to eliminate the twist produced by this torque, it is necessary to displace the plane of loading parallel to itself from the position passing through the X axis by the amount D, shown in Fig. 4, where

$$D = \frac{T}{V} \tag{9}$$

$$V = \frac{\mathrm{d}M}{\mathrm{d}x}. \tag{10}$$

For the shearing force V we have

$$V = \frac{V_z}{\cos\theta} = \frac{\mathrm{d}M_y/\mathrm{d}x}{\cos\theta}. \tag{11}$$

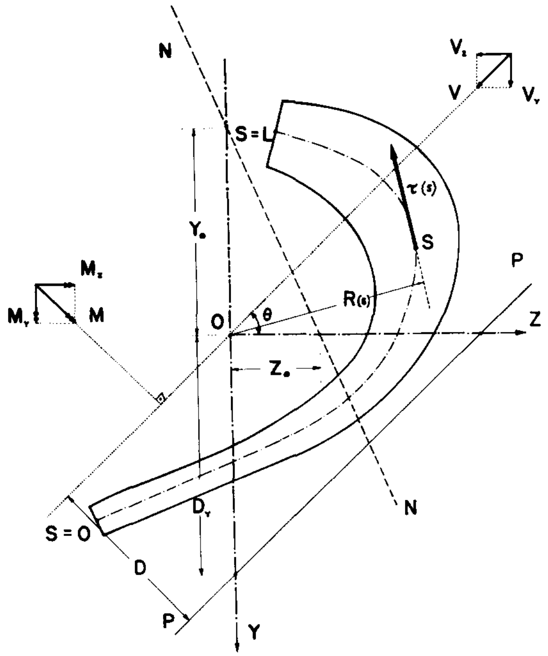


FIG. 4.

At this stage it is useful to consider a transformation of equation (8). By using the integration by parts formula given by equation (12)

$$\int_{s=0}^{s=L} u\,\mathrm{d}v = [uv]_0^L - \int_{s=0}^{s=L} v\,\mathrm{d}u \tag{12}$$

equation (8) can be written as follows:

$$\begin{aligned}\int_{s=0}^{s=L} R(s)\int_0^s B(s)\frac{\partial\sigma(s)}{\partial x}\,\mathrm{d}s\,\mathrm{d}s &= \left[sR(s)\int_0^s B(s)\frac{\partial\sigma(s)}{\partial x}\,\mathrm{d}s\right]_0^L - \int_{s=0}^{s=L} sR(s)B(s)\frac{\partial\sigma(s)}{\partial x}\,\mathrm{d}s \\ &\quad - \int_{s=0}^{s=L} s\frac{\mathrm{d}R(s)}{\mathrm{d}s}\int_0^s B(s)\frac{\partial\sigma(s)}{\partial x}\,\mathrm{d}s\,\mathrm{d}s\end{aligned} \tag{13}$$

where we used for u and v the equations (14)

$$\begin{aligned} u &= R(s)\int_{s=0}^{s=L} B(s)\frac{\partial\sigma(s)}{\partial x}\,\mathrm{d}s \\ v &= s. \end{aligned} \tag{14}$$

Now, from equation (4) it follows that

$$\begin{aligned}\left[sR(s)\int_0^s B(s)\frac{\partial\sigma(s)}{\partial x}\,\mathrm{d}s\right]_0^L &= -\left[sR(s)\int_0^s \frac{\partial}{\partial x}[B(s)\tau(s)]\,\mathrm{d}s\right]_0^L \\ &= -[sR(s)B(s)\tau(s)]_0^L = -LR(L)B(L)\tau(L) = 0\end{aligned} \tag{15}$$

since $\tau(L) = 0$.

Therefore, equation (8) may now be written as

$$T = \int_{s=0}^{s=L} sR(s)B(s)\frac{\partial\sigma(s)}{\partial x}\,\mathrm{d}s + \int_{s=0}^{s=L} s\frac{\mathrm{d}R(s)}{\mathrm{d}s}\int_0^s B(s)\frac{\partial\sigma(s)}{\partial x}\,\mathrm{d}s\,\mathrm{d}s. \tag{16}$$

When $R(s)$ is a stepwise constant function, the second integral in (16) vanishes and the expression of the torque simplifies to

$$T = \int_{s=0}^{s=L} R(s)B(s)\frac{\partial\sigma(s)}{\partial x}s\,\mathrm{d}s. \tag{17}$$

We now consider the three equations of statics which give

$$\int_{s=0}^{s=L} B(s)\sigma(s)\,\mathrm{d}s = 0 \tag{18}$$

$$\int_{s=0}^{s=L} B(s)\sigma(s)y\,\mathrm{d}s = -M_z \tag{19}$$

$$\int_{s=0}^{s=L} B(s)\sigma(s)z\,\mathrm{d}s = M_y \tag{20}$$

where M_z and M_y are the bending moments with respect to the Z and Y axes respectively. The direction of the moment vector is determined by the ratio

$$\frac{M_z}{M_y} = \tan\theta \tag{21}$$

where θ is the angle of the moment vector with the Y axis.

Introducing the distance $D_y = D/\cos\theta$, one may rewrite equation (9) as

$$D_y = \frac{T}{\mathrm{d}M_y/\mathrm{d}x} = \frac{\int_{s=0}^{s=L} R(s)\int_0^s B(s)[\partial\sigma(s)/\partial x]\,\mathrm{d}s\,\mathrm{d}s}{\int_{s=0}^{s=L} B(s)[\partial\sigma(s)/\partial x]z\,\mathrm{d}s}. \tag{22}$$

If (Y^*, Z^*) is a generic point of the straight line PP the equation of the line PP is given by

$$D_y - Y^* = \tan\theta\,.\,Z^* \tag{23}$$

which with equations (19), (20) and (22) can be written in the form

$$\frac{\int_{s=0}^{s=L} R(s)\{\int_0^s B(s)[\partial\sigma(s)/\partial x]\,\mathrm{d}s\}\,\mathrm{d}s}{\int_{s=0}^{s=L} B(s)[\partial\sigma(s)/\partial x]z\,\mathrm{d}s} - Y^* = -\frac{\int_{s=0}^{s=L} B(s)\sigma(s)y\,\mathrm{d}s}{\int_{s=0}^{s=L} B(s)\sigma(s)z\,\mathrm{d}s}. \tag{24}$$

For a given cross section the parameters defining the position of the straight line in equation (24) are those of equation (2); that is $\dot{\alpha}$, $\dot{\beta}$, and $\dot{\gamma}$.

Suppose that equation (3) has the form of a power law (1) where A and n are material constants depending generally only on the temperature at each point of the cross section. This power law has been used extensively in the literature of creep. Figure 5 gives a graphical representation of this law for several values of n. It is seen that for $n = 1$ we have linear creep, while for $n = 0$ the rate of strain is indeterminate at $\sigma = A$ and zero at $\sigma < A$. The elastic analog, see Hoff [10] states that $n = 1$ corresponds to elasticity and $n = 0$ to plasticity.

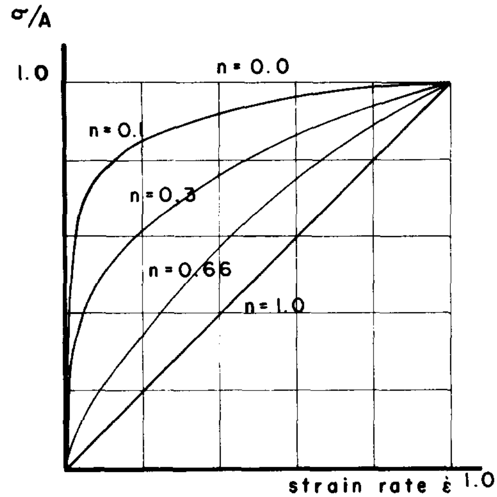


FIG. 5.

With equation (3), when $\dot{\gamma} \neq 0$, the equation becomes

$$\sigma = (\text{sign}\,\dot{\varepsilon})A\dot{\gamma}^n|ay+bz+1|^n \tag{25}$$

where

$$a = \frac{\dot{\alpha}}{\dot{\gamma}}, \qquad b = \frac{\dot{\beta}}{\dot{\gamma}}. \tag{26}$$

Figure 4 shows that the position of the neutral axis is defined by the two points with coordinates $(Y_0, 0)$ and $(0, Z_0)$, which are intersections of the neutral axis with the Y and Z axis respectively. We easily obtain

$$a = -\frac{1}{Y_0}, \qquad b = -\frac{1}{Z_0}. \tag{27}$$

Using equation (25), equations (18) and (21) can be expressed as follows:

$$\int_{s=0}^{s=L} B(s)(\text{sign}\,\dot{\varepsilon})|ay+bz+1|^n\,\mathrm{d}s = 0 \tag{28}$$

$$-\frac{\int_{s=0}^{s=L} B(s)(\text{sign}\,\dot{\varepsilon})|ay+bz+1|^n y\,\mathrm{d}s}{\int_{s=0}^{s=L} B(s)(\text{sign}\,\dot{\varepsilon})|ay+bz+1|^n z\,\mathrm{d}s} = \tan\theta. \tag{29}$$

From equation (28) we can see that for a given value of a the corresponding value of b is uniquely determined and, therefore, that the position of the neutral axis is a function of only one parameter. The one to one correspondence between the lines PP and NN shows that also the lines PP depend on one parameter only. Our task will be to obtain their envelope by eliminating this parameter.

From equations (28) and (29) it is seen that the position of the neutral axis is independent of the magnitude of the bending moment; it depends on the exponent n, but not on the coefficient A, since this latter coefficient has been eliminated from both equations. For a given θ equations (28) and (29) give a solution for a and b. This solution is independent of the bending moment, and consequently of x. From equation (25) we can obtain, therefore

$$\frac{\partial \sigma}{\partial x} = \frac{\partial}{\partial x}(\dot{\gamma}^n)A|ay+bz+1|^n \tag{30}$$

and from equations (24) and (30) we obtain

$$\frac{\int_{s=0}^{s=L} R(s)[\int_0^s B(s)(\operatorname{sign}\dot{\varepsilon})|ay+bz+1|^n\,\mathrm{d}s]\,\mathrm{d}s}{\int_{s=0}^{s=L} B(s)(\operatorname{sign}\dot{\varepsilon})|ay+bz+1|^n z\,\mathrm{d}s} - Y^* = -\frac{\int_{s=0}^{s=L} B(s)|ay+bz+1|^n y\,\mathrm{d}s}{\int_{s=0}^{s=L} B(s)|ay+bz+1|^n z\,\mathrm{d}s}Z. \tag{31}$$

By differentiating equations (28) and (31) with respect to the parameter a, and using equation

$$\frac{\partial}{\partial a}[(\operatorname{sign}\dot{\varepsilon})|ay+bz+1|^n] = ny(ay+bz+1)^{n-1} \tag{32}$$

we obtain the following equations:

$$\int_{s=0}^{s=L} B(s)y(ay+bz+1)^{n-1}\,\mathrm{d}s + \frac{\mathrm{d}b}{\mathrm{d}a}\int_{s=0}^{s=L} B(s)z(ay+bz+1)^{n-1}\,\mathrm{d}s = 0 \tag{33}$$

$$\begin{aligned} &Y^*\left[\int_{s=0}^{s=L} Yz(ay+bz+1)^{n-1}B(s)\,\mathrm{d}s + \frac{\mathrm{d}b}{\mathrm{d}a}\int_{s=0}^{s=L} z^2B(s)(ay+bz+1)^{n-1}\,\mathrm{d}s\right] \\ &\quad - Z^*\left[\int_{s=0}^{s=L} y^2(ay+bz+1)^{n-1}B(s)\,\mathrm{d}s + \frac{\mathrm{d}b}{\mathrm{d}a}\int_{s=0}^{s=L} B(s)yz(ay+bz+1)^{n-1}\,\mathrm{d}s\right] \\ &= -\int_{s=0}^{s=L} B(s)R(s)y(ay+bz+1)^{n-1}s\,\mathrm{d}s - \frac{\mathrm{d}b}{\mathrm{d}a}\int_{s=0}^{s=L} B(s)R(s)(ay+bz+1)^{n-1}s\,\mathrm{d}s. \end{aligned} \tag{34}$$

In principle, the equations of the envelope of the lines PP will be obtained by eliminating the parameters a and b between equations (28), (31) and (34), where $\mathrm{d}b/\mathrm{d}a$ has already been eliminated by using equation (33). Due to the nonlinearity of these equations the envelope is constructed by obtaining the contact points $(\bar{Y}, \bar{Z})$ between the envelope and the lines PP. These contact points are determined by solving equations (31) and (34).

APPLICATION TO A SEMICIRCULAR CROSS SECTION

We apply the results of the previous discussion to a semicircular cross section as shown in Fig. 6. The origin of the coordinates is such that

$$z = R\cos\omega \qquad y = R\sin\omega \qquad -\frac{\pi}{2} \le \omega \le \frac{\pi}{2}. \tag{35}$$

In this particular cross section we selected $R = 10$ in.; the results of this section will be valid, however, for any value of R. Our results are independent of the selected value of B since B is constant throughout the cross section.

Figure 7 gives the coordinate Z_0 as a function of the angle ϕ which the neutral axis NN makes with the coordinate axis Y. Here Z_0 is the coordinate of the intersection point

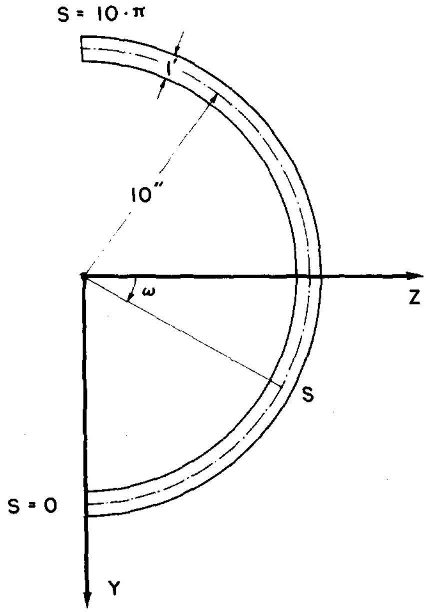


FIG. 6.

of NN with the Z axis. The function $Z_0 = f(\phi)$ is given for several values of the exponent n. It is seen that Z_0 is practically constant between $\phi = 1{\cdot}0$ and $\phi = \pi/2$. For $n = 1$ all neutral axes pass through the centroid. We also notice that for decreasing values of n the range of the values Z_0 is increasing.

In Fig. 8 we see the envelopes of the neutral axes for several values of n which have been obtained by a method explained in Ref. [5]. In Fig. 9 we see the envelopes of the torsionfree axes for the same values of n.

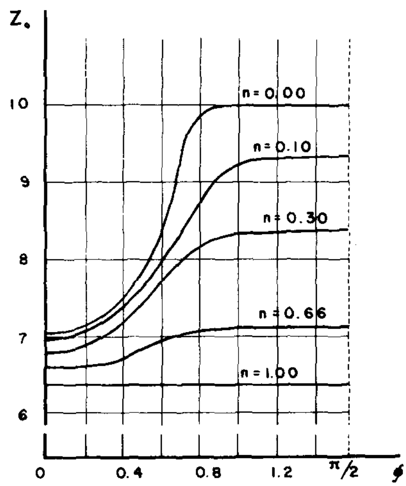
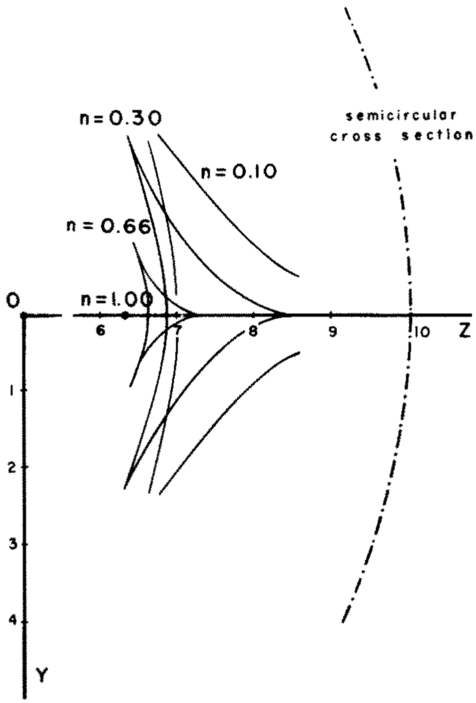
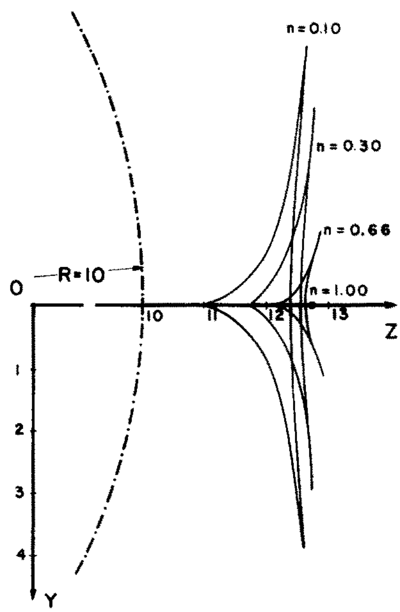


FIG. 7.



n = 0.30
n = 0.10
n = 0.66
n = 1.00
semicircular cross section
O
6
7
8
9
10
Z
1
2
3
4
Y

FIG. 8.



n = 0.10
n = 0.30
n = 0.66
n = 1.00
R = 10
O
10
11
12
13
Z
1
2
3
4
Y

FIG. 9.

The contact points between the envelopes and the lines PP were determined by solving equations (31) and (34). These equations could be simplified considerably since $R(s)$ is in the present case a constant. The same simplification is valid when $R(s)$ is a stepwise function. Indeed, by using equation (17), equation (31) becomes

$$Y^* \int_{s=0}^{s=L} B(s)(\text{sign}\, \dot{\varepsilon})|ay+bz+1|^n z\, ds - Z^* \int_{s=0}^{s=L} B(s)(\text{sign}\, \dot{\varepsilon})|ay+bz+1|^n y\, ds$$

$$= \int_{s=0}^{s=L} B(s)R(s)(\text{sign}\, \dot{\varepsilon})|ay+bz+1|^n s\, ds \tag{36}$$

and equation (34) becomes

$$Y^*\left[\int_{s=0}^{s=L} yz(ay+bz+1)^{n-1}B(s)\, ds + \frac{db}{da}\int_{s=0}^{s=L} z^2(ay+bz+1)^{n-1}B(s)\, ds\right]$$

$$-Z^*\left[\int_{s=0}^{s=L} y^2(ay+bz+1)^{n-1}B(s)\, ds + \frac{db}{da}\int_{s=0}^{s=L} yz(ay+bz+1)^{n-1}B(s)\, ds\right]$$

$$= \int_{s=0}^{s=L} R(s)y(ay+bz+1)^{n-1}B(s)s\, ds + \frac{db}{da}\int_{s=0}^{s=L} R(s)z(ay+bz+1)^{n-1}B(s)s\, ds. \tag{37}$$

These two equations are much simpler than equations (31) and (34), respectively.

Inspection of Fig. 9 shows that the envelopes of the torsionfree axes are curvilinear triangles which for decreasing n grow in size and are displaced in the direction of the cross section. These curvilinear triangles correspond to the curvilinear triangles in Fig. 8 representing the envelopes of the neutral axes.

For $n = 1$ the curvilinear triangle in Fig. 9 is reduced to the shear center of elasticity. A change of n from 1 to 0·1 requires a parallel displacement e of the line PP from 0·25 to 1·10 in., depending on the orientation θ of PP, see Fig. 10. Even a change of n from 0·66 to 0·30 requires a value e from 0·3 to 0·5 in.

Table 1 gives the necessary displacements e for changes of n from 1·0 to values 0·10, 0·30, and 0·66 for several angles θ.

From data presented in Odqvist [8] and in Hult [9] and shown graphically in Fig. 11 it is shown that the exponent n is a function of the temperature and that it changes considerably with increasing temperature. Typical changes are for copper from 0·2 at 370°F to 0·52 at 450°F, and for carbon steel from 0·15 at 750°F to 0·35 at 1020°F. We

TABLE 1

θ (°)	$n = 0{\cdot}10$ (in.)	$n = 0{\cdot}30$ (in.)	$n = 0{\cdot}66$ (in.)
20	0·504	0·354	0·154
30	0·67	0·47	0·21
40	0·86	0·60	0·26
50	0·94	0·67	0·27
60	1·00	0·70	0·26
80	0·69	0·39	0·15
90	0·10	0·25	0·13

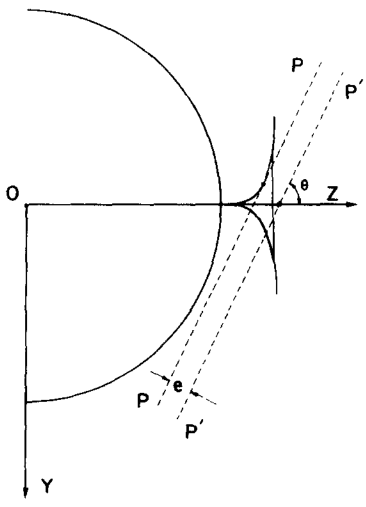


FIG. 10.

conclude that a relatively small change in temperature may produce a substantial change in the value of n and consequently a substantial displacement of the line PP. If not effected, this displacement will produce torsional shearing stresses in the cross section in addition to the already existing bending shearing stresses. In what follows, it will be seen that these additional torsional shearing stresses are of the same order of magnitude as the original bending shearing stresses.

Even without a change in the temperature a rotation of the plane of bending requires an appropriate translation of this plane in order for the line PP to remain tangent to the envelope. Assume that for some value of n the intersection of PP in its vertical position with the Z axis is fixed and that rotation of PP can be effected only about the intersection point. Then it is seen that if $n = 0{\cdot}1$ a 45° rotation of the line PP requires a displacement of approximately 0·75 in., a value which corresponds to substantial additional torsional shearing stresses.

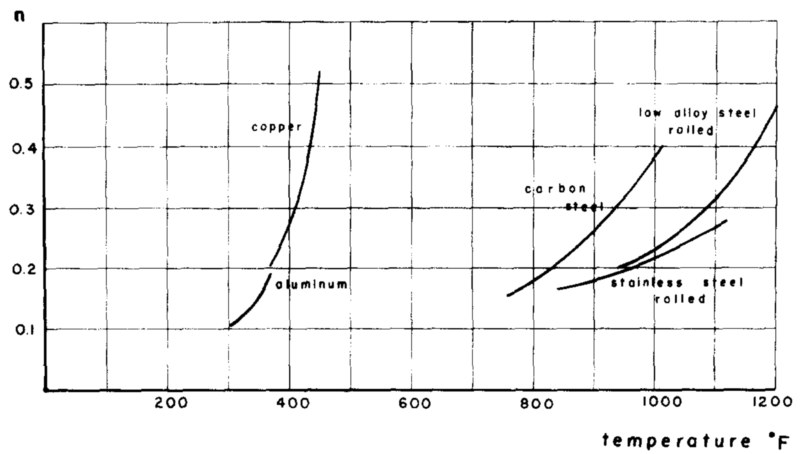


FIG. 11.

We shall now calculate the shearing stresses due to bending in the absence of torsion and then we shall compute the additional shearing stresses which will be produced by the torsion caused by an arbitrary parallel displacement of the plane PP.

The shearing stresses due to bending are given by equation (6) which with equation (25) and for a constant B becomes

$$\tau = -A\frac{\partial|\dot{\gamma}|^n}{\partial x}\int_0^s (\operatorname{sign}\dot{\varepsilon})|ay+bz+1|^n \,\mathrm{d}s. \tag{38}$$

For given values of a and b, i.e., for a given direction of the loading plane, the angle θ which the loading plane makes with the z axis is given by equation (29) written now as

$$\tan\theta = -\frac{\int_{s=0}^{s=L}(\operatorname{sign}\dot{\varepsilon})|ay+bz+1|^n y\,\mathrm{d}s}{\int_{s=0}^{s=L}(\operatorname{sign}\dot{\varepsilon})|ay+bz+1|^n z\,\mathrm{d}s}. \tag{39}$$

The contribution to the shearing force V of the shearing stress τ at any value s, or angle ω, is

$$\tau_v = \tau\sin(\theta-\omega) \tag{40}$$

so that we now write

$$V = \int_{s=0}^{s=L}\tau_v B\,\mathrm{d}s = ABR\frac{\partial|\dot{\gamma}|^n}{\partial x}\int_{\omega=-\pi/2}^{\omega=\pi/2}(\operatorname{sign}\dot{\varepsilon})|ay+bz+1|^n\sin(\theta-\omega)\,\mathrm{d}\omega. \tag{41}$$

Thus we obtain

$$\frac{\tau}{V} = \frac{\int_{\omega=-\pi/2}^{\omega=\pi/2}(\operatorname{sign}\dot{\varepsilon})|ay+bz+1|^n\,\mathrm{d}\omega}{B\int_{\omega=-\pi/2}^{\omega=\pi/2}(\operatorname{sign}\dot{\varepsilon})|ay+bz+1|^n\sin(\theta-\omega)\,\mathrm{d}\omega}. \tag{42}$$

In Fig. 12 we show the distribution of τ/V along the cross section for $n = 0{\cdot}100$ at a specific position of the neutral axis. The values are given in terms proportional to $1/B$. The shearing stresses due to torsion caused by an arbitrary parallel displacement e of the plane PP will be obtained easily for the two limiting cases $n = 1$ and $n = 0$. For $n = 1$ the shearing stress distribution across the width of the cross section is linear. We assume that this distribution is also independent of ω.†

Then, using expressions given in Wang [11] we obtain

$$\frac{\tau_{\max}^{el}}{V} = \frac{3e}{LB^2}. \tag{43}$$

For $n = 0$ the stress distribution is perfectly plastic so that we obtain

$$\frac{\tau_{\max}^{pl}}{V} = \frac{4e}{LB^2}. \tag{44}$$

For intermediate values of n the ratio $\tau_{\max}/V$ is

$$\frac{3e}{LB^2} \le \frac{\tau_{\max}}{V} \le \frac{4e}{LB^2}. \tag{45}$$

† This assumption is correct with very good approximation except in the neighborhood of $\omega = \pm\,\pi/2$.

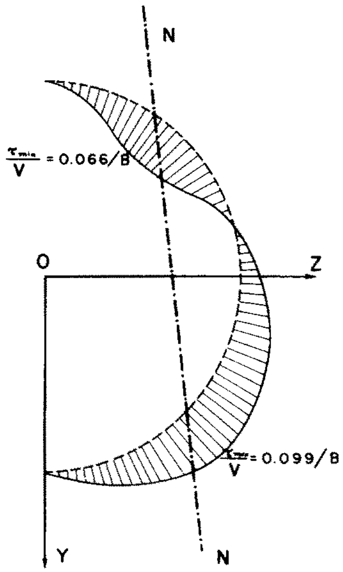


FIG. 12.

To compare the values of τ/V due to torsion with the values due to bending we assume as a first approximation that τ_{max}/V changes linearly with n between $n = 1$ and $n = 0$.

We now remark that $(\tau_{max}/V)_{torsion}$ is inversely proportional to B^2 whereas $(\tau_{max}/V)_{bending}$ is inversely proportional to B. Then, for a sufficiently small B, $(\tau_{max}/V)_{torsion}$ can be made equal or larger than $(\tau_{max}/V)_{bending}$. Assuming, for example, that $B = 1, n = 0{\cdot}1, L = 10\pi$ in., $\theta = 30°$, from Table 1 we obtain $e = 0{\cdot}67$ in., and from inequalities (45) we obtain $(\tau_{max}/V)_{torsion}$ while from Fig. 12 the value of $(\tau_{max}/V)_{bending}$ is found. We see that $(\tau_{max}/V)_{bending} = 0{\cdot}0993$ lb/in.^{2} while $(\tau_{max}/V)_{torsion} = 0{\cdot}0832$ lb/in.^{2}. These two values are of the same order of magnitude. We thus observe that the determination of the envelopes of the torsionfree axes is necessary for a correct determination of the carrying capacity of a thin-walled beam of open cross section.

CONCLUDING REMARKS

By using the elementary theory of bending of beams, the concept of the shear center for straight thin-walled beams of open cross section has been generalized to the case of creep. It was shown that the shear center is replaced by the envelope of a family of straight lines—the torsion-free axes—such that when the plane of bending passes through such a straight line the cross section will be free of torsion.

For a power type stress–strain rate creep law

$$\sigma = \operatorname{sign} \dot{\varepsilon}\, A|\dot{\varepsilon}|^n$$

the envelope was shown to depend on the exponent n which varies with the temperature. Hence a change in the temperature requires a translation of the plane of loading to avoid torsion. But even without a change in temperature a rotation of the plane of bending must be accompanied by an appropriate translation to avoid torsion because the torsion-free axes do not meet at one point. It was shown that omission of the translation of the plane of loading introduces torsional shearing stresses of an order of magnitude equal to the bending stresses. Hence this translation cannot be neglected in design.

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Абстракт—Работа обобщает для проблемы ползучести понятие центра сдвига для прямолинейных тонкостенных балок, открытого профиля. Указывается, что центр сдвига заменяется огибающей семейства прямолинейных, свободных от кручения осей так, что когда плоскость изгиба переходит сквозь такой прямой линии, поперечное сечение не подвергается кручению. Дается метод для расчета этой огибающей.

Принимая, что закон скорости деформаций и напряжений является степенного типа, указывается, что эта огибающая представляет собой функцию показателя–степенного закона. Учитывая, что значение–зависит от температуры, тогда изменение температуры требует трансляции плоскости нагрузки, с целбю избежения кручения. Пропуск этой трансляции вызывает добавочное напряжение сдвига, благодаря кручению, которого порядок величины равен напряжению сдвига вследствие изгиба. Теория применяется к полукруглому поречному сечению. Расчеты проведены на цифровой машине IBM 7094–7040.